Problem of the week

Nuclear fusion

(a) Outline how a main sequence star maintains equilibrium.
(b) State the common characteristic of all main sequence stars.
(c) Suggest how nuclear fusion in stellar cores is possible.

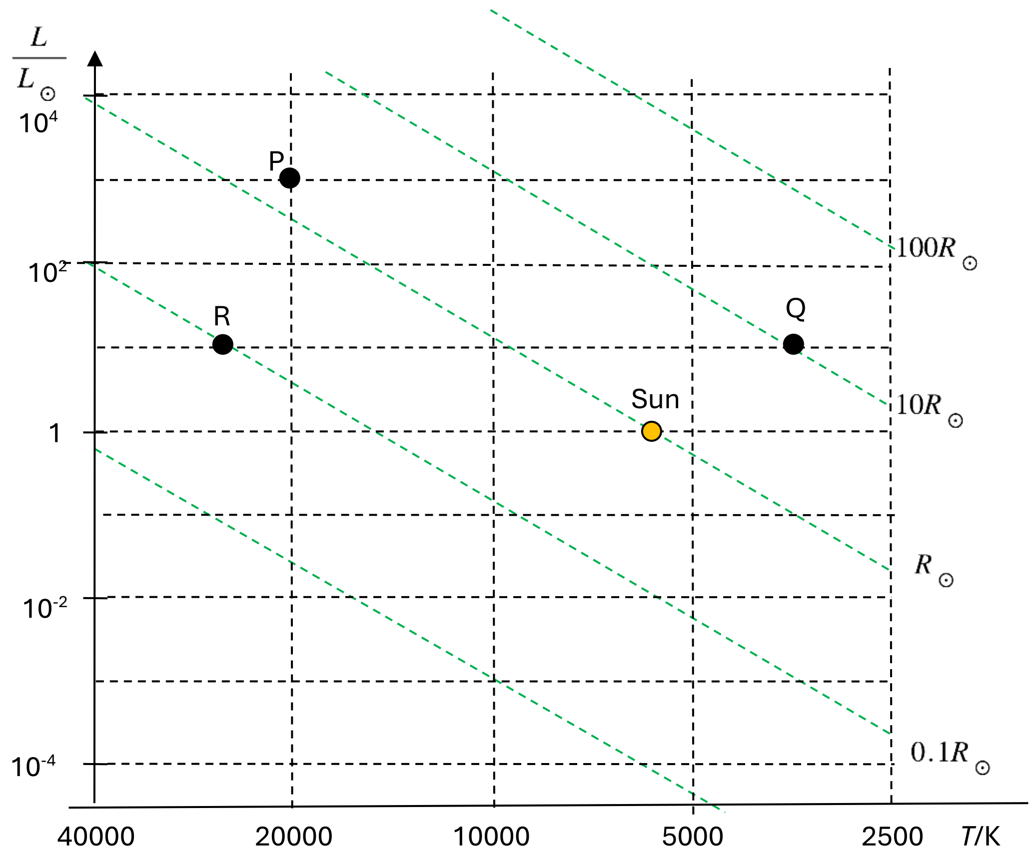
(d) A possible fusion reaction in stars is ${}^{3}_{2}\mathrm{He} + {}^{6}_{3}\mathrm{Li} \to {}^{2}_{1}\mathrm{H} + {}^{7}_{4}\mathrm{Be}$. The following atomic masses are available:

He	3.016029 u
Li	6.015123 u
H	2.014102 u
Be	7.016930 u

Calculate the energy released.

(e) The simplest fusion reaction in stars is ${}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \to {}^{2}_{1}\mathrm{H} + e^{+} + \nu$. This reaction requires temperatures of order 10^{7} K.
(i) Suggest why the reaction in (d) requires higher temperatures than this reaction.
(ii) (HL only) Estimate this higher temperature.
(iii) Suggest why iron is the heaviest element that can be produced in stellar cores.

(f) The diagram shows an incomplete HR diagram.



(i) The radius of the Sun is 7.0×10^8 m and its luminosity is 3.8×10^{26} W. Show that the surface temperature of the Sun is about 6000 K.

(ii) Determine the radius of star P.

(iii) The mass of P is about 7 solar masses. Outline the likely evolution of star P.

(iv) Determine the ratio of the surface temperature of star R to that of star Q.

Answers

(a) The inward gravitational pressure due to gravitational forces is balanced by the outward thermal/radiation pressure produced by the outflow of energy due to fusion in the stellat core.

(b) They fuse hydrogen into helium.

(c) $\Delta m = 3.016029 + 6.015123 - (2.014102 + 7.016930) = 1.20\times 10^{-4}$ u. So, $Q = \Delta m \times c^2 = 1.20\times 10^{-4} \times 931.5 = 0.112$ MeV.

(d) Stellar cores exist at high temperatures and high densities satisfying the two conditions for fusion to occur. The high temperatures enable nuclei to move fast and hence crash into each other leading to fusion. The high densities ensure that enough fusion reactions take place for lots of energy to be produced.

(e)

(i) It is higher because the Coulomb barrier that needs to be overcome is higher.

(ii) (HL only) The barrier is 6 times higher, so an estimate is 6×10^7 K.

(iii) Iron is near the peak of the binding energy curve, so heavier elements are not energetically possible to be produced.

(f)

(i) $L = \sigma A T^4$ so $T^4 = \dfrac{L}{4\pi\sigma R^2} \Rightarrow T = \left(\dfrac{L}{4\pi\sigma R^2}\right)^{\frac{1}{4}} = \left(\dfrac{3.8\times 10^{26}}{4\pi \times 5.67\times 10^{-8} \times (7.0\times 10^8)^2}\right)^{\frac{1}{4}} = 5744$ K.

(ii) $\dfrac{L}{L_\odot} = 10^3 = \dfrac{R_P^2 T_P^4}{R_\odot^2 T_\odot^4}$ so $\dfrac{R_P^2}{R_\odot^2} = 10^3 \times \dfrac{T_\odot^4}{T_P^4} = 10^3 \times \dfrac{5744^4}{20000^4}$. Hence $\dfrac{R_P^2}{R_\odot^2} = 6.80 \quad \Rightarrow \quad R_P = 2.6 R_\odot = 1.8\times 10^9$ m.

(iii) The most likely evolution of star P is a planetary nebula after which it will become a white dwarf leaving the core (of carbon) with a mass under the Chandrasekhar limit.

(iv) R and Q have the same luminosity so $\sigma 4\pi R_R^2 T_R^4 = \sigma 4\pi R_Q^2 T_Q^4$. Hence $\dfrac{T_R^4}{T_Q^4} = \dfrac{R_Q^2}{R_R^2} = \left(\dfrac{10}{0.1}\right)^2 = 10^4$ and so $\dfrac{T_R}{T_Q} = 10$.